Making Sense of the Experimental Evidence on Endogenous Timing in Duopoly Markets†

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Abstract

The prediction of asymmetric equilibria with Stackelberg outcomes is clearly the most frequent result in the endogenous timing literature. Several experiments have tried to validate this prediction empirically, but failed to find support for it. By contrast, the experiments find that simultaneous-move outcomes are modal and that behavior in endogenous timing games is quite heterogeneous. This paper generalizes Saloner's (1987) and Hamilton and Slutsky's (1990) endogenous timing games by assuming that players are averse to inequality in payoffs. We explore the theoretical implications of inequity aversion and compare them to the empirical evidence. We find that this explanation is able to organize most of the experimental evidence on endogenous timing games. However, inequity aversion is not able to explain delay in Hamilton and Slutsky's endogenous timing games.

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1 Introduction

The theoretical literature on endogenous timing started with Saloner (1987), Hamilton and Slutsky (1990), and Robson (1990). This literature tries to identify factors that might lead to the endogenous emergence of sequential or simultaneous play in oligopolistic markets.

Saloner (1987) analyzes a duopoly with two periods where firms can produce in *both* periods before the market clears. In the first period firms simultaneously choose initial production levels. The choices of the first period are observed and then additional non-negative second period outputs are chosen simultaneously. Saloner shows that if production costs are the same across both periods, then there is a continuum of equilibria: any point on the outer envelope of the reaction functions between the firm's Stackelberg outputs is attainable with a subgame perfect Nash equilibrium (SPNE). Additionally, in all of these equilibria production takes place only in the first period. However, Ellingsen (1995) shows that only the two Stackelberg equilibria in Saloner's game survive elimination of weakly dominated strategies.[1]

In Hamilton and Slutsky (1990)'s action commitment game, two firms must decide a quantity to be produced in *one* of two periods before the market clears. If a firm commits to a quantity in the first period, it acts as the leader but it does not know whether the other firm has chosen to commit early or not. If a firm commits to a quantity in the second period, then it observes the first period production of the opponent (or its decision to wait). Hamilton and Slutsky show that this game has three SPNE: both firms committing in the first period to the simultaneous-move Cournot-Nash equilibrium quantities, and each waiting and the other playing its Stackelberg leader quantity in the first period. They also show that only the Stackelberg equilibria survive elimination of weakly dominated strategies.[2]

Observed behavior in experiments on these two canonical models of endogenous timing is at odds with the theory. For example, Huck et al. (2002) test experimentally the predictions of Hamilton and Slutsky (1990)'s action commitment game. They find that: (i) Stackelberg outcomes are rare, (ii) simultaneous-move Cournot outcomes are modal, (iii) simultaneous-move outcomes are often played in the second production period, and (iv) behavior is quite heterogeneous–in some cases followers punish leaders, in other cases collusive outcomes are played, and in other cases Stackelberg warfare is observed.[3] Müller (2006) tests the predictions of Saloner's game extended by Ellingson. He finds that: (i) Stackelberg outcomes are extremely rare, (ii) simultaneous-move

[1] Several papers have suggested ways to reduce the set of equilibria in Saloner's model by modifying the structure of the game. For example, Robson (1990) introduces discounting between periods, Pal (1991) introduces cost asymmetries between periods, Maggi (1996) introduces uncertainty about demand.

[2] A model where the price is chosen was considered by Robson (1990), and a Stackelberg outcome is also obtained.

[3] Throughout the paper we consider that collusive outcomes describe situations where both firms produce less than their Cournot-Nash quantities. We also consider that Stackelberg warfare describes a situation where both firms produce more than their Cournot-Nash outputs.

symmetric outcomes are the most frequent outcomes, (iii) sometimes collusive outcomes are observed, (iv) there is production in both periods with 84% of production taking place in the first period, (v) subjects seem to attempt to balance market shares in the second production period, and (vi) subjects do not produce more than the Stackelberg follower's quantity in the first production period.[4]

The questions that the endogenous timing literature tries to address are particularly relevant in terms of new markets, where two or more firms will enter. The experimental evidence suggests that simultaneous-move play may a better predictor of behavior in markets for new goods than sequential play.[5] It also suggests that there may be substantial heterogeneity in behavior in these markets. In some cases collusive outcomes may emerge, in other cases Stackelberg warfare, and in others still sequential play with Stackelberg like outcomes.[6]

Why does the theory perform poorly in the experiments? One possibility is that subjects are not able to iteratively rule out dominated strategies and stop after one or two rounds of reasoning. There is substantial experimental evidence that supports this view. Even if subjects are able to do eliminate dominated strategies the two Stackelberg equilibria involve large payoff differences and this creates a coordination problem. This implies that playing the Stackelberg leader's quantity is risky by comparison with playing the Cournot-Nash quantity.[7]

It is possible to think of explanations for some aspects of the empirical evidence. However, it is much harder to explain all of the experimental findings. For example, the risk-payoff equilibrium selection argument may explain why simultaneous-move outcomes are more frequently played than Stackelberg outcomes. However, it cannot explain the emergence of collusive outcomes or Stackelberg warfare. It is also not clear how this explanation can account for the fact that simultaneous-move play can take place in the second production period in Hamilton and Slutsky's action commitment game.

The gap between the theory and the experimental evidence is the main motivation behind this paper. To bridge this gap the paper assumes that players in endogenous timing games have social preferences. The paper derives the predictions of this explanation for both Saloner's and Hamilton and Slutsky's endogenous timing games and compares the predictions to the empirical evidence.

Social preferences have been shown to explain a broad range of data for many different games. The clearest evidence for these type of preferences comes from bargaining and trust games. For example, in ultimatum games offers are usually much more generous than predicted by equilibrium and low offers are

[4] Section 2 discusses the experimental evidence on endogenous timing games.

[5] As we have seen the prediction of Stackelberg equilibria rests on equilibrium selection argumens. Simultaneous-move Cournot-Nash equilibria typically exist, however, they do not survive the application of equilibrium refinements.

[6] Bagwell (1995) points out that the theoretical prediction of Stackelberg outcomes crucially depends on the perfect observability of the Stackelberg leader's action. However, the experiments assume perfect observability which rules out this explanation.

[7] See Harsanyi and Selten's (1988) for a discussion of risk-payoff dominance considerations.

often rejected. According to the social preferences explanation, these offers are consistent with an equilibrium in which players make offers knowing that other players may reject allocations that appear unfair. Huck et al. (2002), Müller (2006), and Fonseca et al. (2005b) suggest that inequity aversion may also explain behavior in endogenous timing games. However, these papers do not formalize this explanation.

The paper starts by generalizing Saloner's (1987) game and Hamilton and Slutsky's (1990) action commitment game by assuming that players are averse to inequality in payoffs. To incorporate inequity aversion in endogenous timing games we make use of Fehr and Schmidt's (1999) approach. That is, we assume that an inequity averse player dislikes advantageous inequity–i.e. it feels compassion towards the opponent if the opponent has lower profits–and also dislikes disadvantageous inequity–i.e. it feels envy towards the opponent if the opponent has higher profits.

The paper shows that relatively high levels of inequity aversion rule out asymmetric equilibria in both Saloner's game as well as in Hamilton and Slutsky's action commitment game. In other words, relatively high levels of inequity aversion favor simultaneous-move play over sequential play. The intuition for this result is straightforward. For relatively high levels of inequity aversion, playing leader type outcomes leads to inequity costs which are larger than the material benefits of leadership.[8]

The paper also shows that inequity aversion gives rise to a continuum of symmetric equilibria in both Saloner's game as well as in Hamilton and Slutsky's action commitment game. The intuition for this result is as follows. Suppose that a player knows that his opponent will produce the Cournot-Nash quantity. If this player is averse to inequity, then his best response is to produce also the Cournot-Nash quantity. Producing an output different from the Cournot-Nash quantity reduces the player's material payoff and increases inequity costs. Now, suppose that a player knows that his opponent will produce somewhat less than the Cournot-Nash quantity. If this player is averse to advantageous inequity, then his best response is to produce exactly the same quantity as the opponent. Producing a higher quantity than the opponent increases the player's material payoff by less than the cost from advantageous inequity. Similarly, if a player knows that his opponent will produce somewhat more than the Cournot-Nash quantity, then his best response is also to produce the same quantity as the opponent. Producing a lower quantity than the opponent increases the player's material payoff by less than the cost from disadvantageous inequity.

The previous paragraph shows us that inequity aversion may lead both players to produce less than the Cournot-Nash quantity. This happens whenever players have a relatively high level of compassion and are able to coordinate on a "collusive outcome." Similarly, inequity aversion may lead both players to produce more than the Cournot-Nash quantity. This happens whenever players have a relatively high level of envy and are unable to coordinate on the

[8] Relatively low levels of inequity aversion do not rule out asymmetric equilibria. In fact, as inequity aversion vanishes the set equilibria of each game converges to the set of equilibria of the respective standard game where players are assumed to care only about material payoffs.

Cournot-Nash equilibrium. Thus, if a population is composed of players with heterogeneous social preferences and these individuals are matched in pairs to play endogenous timing games, then heterogeneity in behavior is to be expected.

It turns out that inequity aversion is able to explain most of the experimental evidence on Saloner's game. As we have seen, inequity aversion can rule out asymmetric equilibria and generates a continuum of simultaneous-move symmetric equilibria. Inequity aversion can also explain the fact that subjects produce in both periods. This happens because inequity aversion gives rise to a multiplicity of symmetric equilibria in the game and subjects have to coordinate on one of them. If subjects are unable to coordinate in one of the multiple symmetric equilibria in the first production period, then they have an incentive to produce in the second production period to attain coordination before the market clears. The lack of coordination on a symmetric outcome in the first period also explains why players act as if they wish to balance market shares in the second production period.

Inequity aversion is also able to explain most experimental findings on Hamilton and Slutsky's action commitment game. Inequity aversion can rule out sequential play and gives rise to a continuum of simultaneous-move symmetric outcomes.[9] Heterogeneity in social preferences across players can explain the diversity of behavior in Hamilton and Slutsky's action commitment game. As we have seen inequity aversion may lead to collusive outcomes and can also generate Stackelberg warfare. Additionally, inequity aversion also explains why followers seem to punish leaders. If inequity aversion is relatively low and there is sequential play, then the leader will feel compassion towards the follower and the follower will feel envious of the leader. A compassionate leader will produce less than a selfish leader and an envious follower will produce more than a selfish follower. This is exactly what the data shows in Huck et al.'s (2002) experiment.

The remainder of this paper is organized as follows. Section 2 reviews the evidence. Section 3 describes Saloner's (1987) and Hamilton and Slutsky's (1990) models and their results. Section 4 extends the models by assuming that players can be averse to inequity and studies the consequences of this assumption. Section 5 summarizes the predictions of the inequity aversion explanation and compares them to the empirical evidence. Section 6 concludes the paper. Proofs of Propositions are in the Appendix.

2 Experimental Evidence

Müller (2006) tests the predictions of Saloner's game extended by Ellingson. In the experiment, fixed pairs of subjects are repeatedly matched to play the game. Upon entering a lab 40 subjects were assigned to a computer and received

[9] Like in Saloner's game with inequity averse plyers, among all the symmetric equilibria in Hamilton and Slutsky's game with inequity averse players, the Cournot-Nash equilibirum may be the one that is more fequently played. This happens because this equilibrium is always a subgame perfect Nash equilibrium of the game no matter if there is inequity aversion or not. That is not the case with the other symmetric equilibria.

written instructions. Subjects could choose quantities from a finite grid between 0 and 100 with .01 as the smallest step. There were two treatments. Treatment "ONE" was a standard one-period Cournot duopoly. Treatment "TWO" was the two-period duopoly game by Saloner. For each treatment 10 markets were conducted. Subjects had all the information about costs and demand and had a profit calculator to try out the consequences of their choices.

For each market there were 25 rounds. After each round was completed subjects were informed about their own quantities and their profits and the quantities of the opponent. The total earnings of each subject were determined by the sum of earnings per round. The earnings of each round were measured in ECU and were determined as the difference between the market price, given by the linear inverse demand function $P = 100 - (q^i + q^j)$, and marginal cost, equal to 1, times total quantity produced, q^i.[10]

It is straightforward to show that in the standard Cournot game with only one production period a unique Nash equilibrium exists and the Cournot-Nash quantity is given by $N = 33$. The Stackelberg leader's quantity is given by $S = 49.50$ and the Stackelberg follower's quantity by $R(S) = 24.75$. The collusive outcome is given by $\left(C^1, C^2\right) = (24.75, 24.75)$. Table I–taken from Müller (2006)–displays average individual quantities in the two production periods along with total individual quantities in both periods again for blocks of rounds separately.

Table I

	1st bloc rds. 1-8	2nd bloc rds. 9-16	3rd bloc rds. 17-24	last rd. rd. 25	all rds.
1st period	**26.27**	**25.71**	**24.86**	**26.90**	**25.66**
	(52.54)	(51.41)	(49.71)	(53.80)	(51.32)
2nd period	**6.31**	**4.76**	**4.91**	**6.30**	**5.37**
	(12.63)	(9.51)	(9.82)	(12.60)	(10.73)
Both periods	**32.58**	**30.46**	**29.77**	**33.20**	**31.03**
	(65.16)	(60.93)	(59.54)	(66.40)	(62.06)

Individual quantities in treatment TWO
Total quantities in parentheses

We see from Table I that a subject produces on average a quantity of 25.66 in the first period. This quantity is very close to the Stackelberg follower's quantity or the collusive quantity of 24.75. We also see that, on average, a subject produces a quantity of 5.37 in the second production period. Thus, there is production in both periods with the bulk of production–namely 83%–taking place in the first production period. We also see that total output is decreasing with experience. This may happen because experience increases collusive outcomes.

[10] At the start of the experiment subjects received a one-time endowment of 500 ECU. At the end of the experiment the sum of earnings per round was converted into DM with 900 ECU =1 DM.

To have a better picture of behavior let us now consider disaggregate data for each of the 10 markets. Table II–taken from Müller (2006)–displays average individual quantities as observed in rounds 17 to 24 in each individual market (ordered according to increasing total output).

Table II

Market	1st period		2nd period		both periods		total
	q_1^1	q_1^2	q_2^1	q_2^2	q^1	q^2	Q
1	**25.00**	**25.00**	**0.00**	**0.00**	**25.00**	**25.00**	**50.00**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
2	**25.00**	**25.00**	**0.00**	**0.00**	**25.00**	**25.00**	**50.00**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
3	**25.00**	**25.00**	**2.38**	**1.00**	**27.38**	**26.00**	**53.38**
	(0.00)	(0.00)	(3.89)	(0.00)	(3.89)	(0.00)	(3.89)
4	**16.13**	**18.75**	**13.13**	**6.25**	**29.25**	**25.00**	**54.25**
	(10.56)	(5.18)	(13.74)	(5.18)	(7.44)	(0.00)	(7.44)
5	**20.00**	**21.00**	**9.25**	**4.88**	**29.25**	**25.88**	**55.13**
	(0.00)	(2.88)	(3.81)	(2.70)	(3.81)	(2.10)	(4.61)
6	**26.25**	**23.38**	**3.50**	**9.00**	**29.75**	**32.38**	**62.13**
	(5.55)	(8.23)	(4.93)	(8.11)	(2.76)	(0.92)	(3.52)
7	**20.00**	**30.00**	**10.00**	**5.00**	**30.00**	**35.00**	**65.00**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
8	**28.00**	**30.00**	**7.38**	**2.50**	**35.38**	**32.50**	**67.88**
	(3.16)	(8.45)	(4.14)	(3.78)	(4.50)	(5.98)	(9.69)
9	**33.13**	**34.63**	**0.00**	**0.50**	**33.13**	**35.13**	**68.25**
	(0.64)	(0.74)	(0.00)	(0.93)	(0.64)	(0.83)	(1.04)
10	**22.88**	**23.00**	**12.25**	**11.25**	**35.13**	**34.25**	**69.38**
	(12.81)	(4.54)	(11.17)	(4.17)	(2.23)	(2.19)	(3.16)

Average quantities in each market in rounds 17-24
Standard deviations in parentheses

Inspection of the last three columns in Table II reveals that, on average, roughly equal market shares are obtained and that there are almost no outcomes that resemble Stackelberg market shares. We also see that markets 1 to 5 display collusive behavior and markets 6 to 10 display Cournot-Nash behavior in the last third of the experiment.

Columns 2 and 3 in Table II show that the average (across 8 rounds) first period quantities produced in the 10 markets are smaller than or equal to the Stackelberg follower's quantity in 14 out of 20 cases, are between the Stackelberg follower's quantity and the Cournot-Nash quantity in 5 out of 20 cases, and are greater than the Cournot-Nash quantity (but not significantly) in only 1 out of 20 cases. Thus, in most cases subjects produce up to the Stackelberg follower's quantity (which is equal to the collusive quantity) in the first production period.[11]

[11] This pattern of behavior is present accross all rounds as we can see in the first row of

Columns 4 and 5 in Table II show that firms do not produce an incremental amount in the second period so that total production is equal to the Cournot-Nash quantities. Also, subjects seem to attempt to balance market shares in the second production period. Subjects who produced more in the first production period produce less in the second production period than subjects who produced more in the first production period. We also see that overproduction, that is, producing outside the outer envelope of the reaction functions, is rarely observed.

Huck et al. (2002) test experimentally the predictions of Hamilton and Slutsky (1990)'s action commitment game. In the experiment they use the linear inverse demand function

$$P(q^1 + q^2) = \max\left\{30 - (q^1 + q^2), 0\right\},$$

and they assume that costs of production are linear and given by $C^i(q^i) = 6q^i$, $i = 1, 2$. According to this specification, the predictions of Hamilton and Slutsky (1990) are as follows. The Stackelberg leader produces in period one the quantity $S = 12$ and the Stackelberg follower produces in period two the quantity $R(S) = 6$. The simultaneous-move Cournot-Nash quantities are played in period one and are given by $\left(N^1, N^2\right) = (8, 8)$. The collusive quantities are $(C^1, C^2) = (6, 6)$. Huck et al. (2002) run an experiment with a large payoff matrix where subjects could pick an integer quantity from 3 to 15 units. They also run an experiment with a small payoff matrix where subjects could select a quantity from the set $\{6, 8, 12\}$. Table III–taken from Huck et al. (2002)–displays the experimental results on an aggregate level for both the large and the small payoff matrices.

Table III

	In period 1	Explicit followers	Both firms in period 2	Total
Large payoff matrix				
Average quantity	9.15	8.93	8.40	17.70
Standard deviation	1.91	1.75	1.67	1.93
Number of observations	543	207	140	890
Small payoff matrix				
Average quantity	8.65	7.89	7.60	16.05
Standard deviation	2.24	1.22	1.21	1.64
Number of observations	136	94	170	400

Table III shows us that, in the experiment with the large payoff matrix, in 543 out of 890 cases (61%) subjects committed themselves in period 1. In the remaining cases subjects decided to wait. Those who decided to produce in the first period produce on average 9.15 units, which is less than the Stackelberg leader's quantity of 12 units. Those who decided to wait and produce in the

Table I.

second period after having observed that the opponent produced in the first period produce an average output of 8.93 units which is larger than the Stackelberg follower's output of 6 units. This seems to imply that Stackelberg followers exhibit aversion to disadvantageous inequity since they are willing to produce more than the material best response to reduce the payoff of the Stackelberg leader. When both subjects decided to wait, 140 out of 890 cases (18%), their average output is 8.40 units, which is similar to the Cournot-Nash quantity. Table III also shows us that, in the experiment with the small payoff matrix, only in 136 out of 400 cases (34%) did subjects commit themselves in the first period. Both subjects decided to wait in 170 out of 400 cases (42%). Average outputs are slightly smaller than those observed with the large payoff matrix.

Huck et al. (2002) also find that explicit followers observed responses in the experiment with the large payoff matrix have a curious pattern. The continuous theoretical best reply function is given by $q^F = 12 - 0.5q^L$. On average, the observed responses of followers have a negative slope when the leaders produce less than 7 units or more than 12 units. However, when leaders produces between 7 and 12 units the responses of followers have a positive slope.[12] Table IV summarizes market outcomes in terms of absolute and relative frequencies for the experiment with the large payoff matrix.

Table IV

Market outcome	Type	Number of cases		Number of cases incl. quant. 9 and 11	
Cournot	Equilibrium	64	14.4%	93	20.9%
Stackelberg	Equilibrium	24	5.4%	33	7.4%
Stackelberg/Cournot	Coord. failure	27	6.1%	41	9.2%
Stackelberg warfare	Coord. failure	21	4.7%	30	6.7%
Stackelberg punished	Other	43	9.7%	55	12.4%
Collusion (successful)	Other	25	5.6%	25	5.6%
Collusion (exploited)	Other	19	4.3%	19	4.3%
Collusion (failed)	Coord. failure	34	7.6%	41	9.2%
Others		188	42.2%	108	24.3%
Sum		445	100%	445	100%

We see from Table IV that the Cournot equilibrium is the most frequent outcome since it represents 14.4% of all outcomes–20.9% of all outcomes when the quantity 9 is counted as a Cournot action. The Stackelberg equilibria occur only rarely since they represent 5.4% of all outcomes–7.4% of all outcomes when the quantity 11 is counted as a Stackelberg leader action. Coordination failure occurs in 10.8% of all outcomes–15.9% when 9 is counted as Cournot and 11 as Stackelberg leader actions. In the experiment with the small payoff matrix Cournot outcomes become much more frequent (45% vs. 20.9%).

[12] See Fig. 2 in Huck et al. (2002). This finding is replicated in Huck et al. (2001) in a game where the roles of leader and follower are exogenously assigned.

The frequencies of successful and unsuccessful collusion are similar than the ones with the large payoff matrix. Coordination failure becomes less important (4.5% vs. 15.9%). Endogenous Stackelberg equilibria occur even less frequently (5% vs. 7.4%) than with the large matrix. The results with the small payoff matrix rule out the possibility that complexity was responsible for the results obtained with the large payoff matrix. Thus, the results with the small payoff matrix reinforce the idea that subjects prefer symmetric Cournot outcomes to asymmetric outcomes.

Fonseca et al. (2005a) show that Huck et al. (2002)'s findings are robust to cost asymmetries.[13] They find that low cost firms are not able to use their cost advantage to become Stackelberg leaders and that Cournot play is modal.[14] Fonseca et al. (2005b) test experimentally Hamilton and Slutsky (1990)' s observable delay game. In this game two firms bindingly announce a production period (one out of two periods) and then produce in the announced sequence. Hamilton and Slutsky show that this game has a unique symmetric equilibrium where firms produce only in the first period. Fonseca et al. (2005b) find that there is delay in players' production decisions.

3 The Model

Following Saloner (1987), consider a symmetric duopoly with two production periods, where the market clears at the end of the second period. In the first production period, firms 1 and 2 simultaneously produce outputs q_1^1 and q_1^2, respectively. These outputs become common knowledge and, in the second production period, the firms simultaneously produce nonnegative outputs q_2^1 and q_2^2. After the second period, price is determined according to the inverse demand function $P(q_1^1 + q_2^1 + q_1^2 + q_2^2)$. The firms choose outputs to maximize expected profits. The firms have the same constant marginal cost of production each period, $c > 0$.

For firm i, define the single-period reaction function[15]

$$R^i(q^j) = \arg_{q^i} \max \left[P\left(q^i + q^j\right) - c\right] q^i,\ i \neq j = 1, 2.$$

We assume that these reaction functions are well behaved.[16] Let $\left(N^1, N^2\right)$ be the unique single-period Cournot-Nash equilibrium outcome. When firm i produces q^i in the first period and firm j produces its best response in the second

[13] We are not aware of any experiment with Saloner's game that allows for cost asymmetries.

[14] Van Damme and Hurkens (1999, 2004) analyze a timing game with cost differences between firms. In their models a unique Stackelberg equilibrium is selected with the most efficient firm being the Stackelberg leader.

[15] The reaction function corrresponding to a standard single production period Cournot model.

[16] By this we mean, $-1 \leq \partial R^i(q^j)/\partial q^i < 0$. The second condition ensures the existence of a unique single-period Cournot-Nash equilibrium. A set of sufficient conditions for R^i functions to be "well-behaved" is that $P(q^i + q^j)$ is strictly positive on some bounded interval $(0, \bar{Q})$ on which it is twice continuously differentiable, strictly decreasing, and concave, with $P(q^i + q^j) = 0$ for $q^i + q^j \geq \bar{Q}$.

period the profit function of firm i is given by

$$\pi_L^i = \left[P\left(q^i + R^j(q^i) \right) - c \right] q^i, \ i \neq j = 1, 2.$$

For simplicity, we assume that only one Stackelberg point exists for each firm. Denote these points by S^i, $i = 1, 2$, with

$$S^i = \arg_{q^i} \max \left[P\left(q^i + R^j(q^i) \right) - c \right] q^i, \ i \neq j = 1, 2.$$

Denote the outer envelope of the firms' reaction functions, R^1 and R^2, by R, and define the set

$$E_S = \left\{ \left(q^1, q^2 \right) : \left(q^1, q^2 \right) \in R, \ q^1 \leq S^1, \text{ and } q^2 \leq S^2 \right\}.$$

Saloner (1987) shows that the following strategies for each firm constitute a subgame perfect Nash equilibrium:

$$q_1^i = a^i, \text{ where } \left(a^1, a^2 \right) \text{ is a point in } E_S$$

and

$$q_2^i = \begin{cases} N^i - q_1^i & \text{if } q_1^i \leq N^i \text{ and } q_1^j \leq N^j \\ 0 & \text{if } q_1^i \geq N^i \text{ and } q_1^j \leq R^j(q_1^i) \\ 0 & \text{if } q_1^i \geq R^i(q_1^j) \text{ and } q_1^j \geq R^j(q_1^i) \\ R^i(q_1^j) - q_1^i & \text{if } q_1^i \leq N^i, \ N^j \leq q_1^j, \text{ and } q_1^i \leq R^i(q_1^j) \end{cases} \tag{1}$$

This result says that any pair of total outputs $\left(q_1^1 + q_2^1, q_1^2 + q_2^2 \right)$ which lies in the outer envelope of the reaction functions between (and including) the firms' Stackelberg outputs is attainable with a SPNE. Furthermore, it also tells us that all production takes place in the first production period. The intuition behind this result is as follows. Let $\left(\bar{a}^1, \bar{a}^2 \right)$ be a point in E_S. Without loss of generality, let $\left(\bar{a}^1, \bar{a}^2 \right)$ be on $R^2(q^1)$. That is, at $\left(\bar{a}^1, \bar{a}^2 \right)$ firm 1's total output is more than it's single-period Cournot output and firm 2's total output equal it's single-period best response to $\bar{a}^1$. According to the above result $\left(\bar{a}^1, \bar{a}^2 \right)$ is a SPNE and the timing of production is $\left\{ \left(q_1^1 = \bar{a}^1, q_2^1 = 0 \right); \left(q_1^2 = \bar{a}^2, q_2^2 = 0 \right) \right\}$. The point $\left(\bar{a}^1, \bar{a}^2 \right)$ is not an equilibrium of the single-period Cournot game since when firm 2 produces $\bar{a}^2$, firm 1 would do better to produce less than $\bar{a}^1$. However, in Saloner's game, firm 1 cannot gain by producing less than $\bar{a}^1$ in the first production period, say by choosing $q_1^1 < \bar{a}^1$. If it does, then firm 2 would produce an additional amount in the second period, $q_2^2 = R^2(q_1^1) - \bar{a}^2 > 0$, and this would lower firm 1's profits below what it gets at $\left(\bar{a}^1, \bar{a}^2 \right)$.

Ellingsen (1995) shows that the two Stackelberg equilibria are the only subgame perfect equilibrium outcomes which survive the elimination of weakly dominated strategies. This happens because producing in the first period is weakly dominated by waiting: if firm 1 is uncertain about which strategy firm 2 is going to play, firm 1's best strategy is to wait, in which case it is able to make a best response to any quantity that firm 2 chooses. But, then this implies that firm 2 would choose the Stackelberg leadership quantity. Elimination of weakly dominated strategies also implies that in the Stackelberg equilibria the leader produces only in the first production period and the follower may produce only in the second period.

In Hamilton and Slutsky's (1990) action commitment game firms can only produce in one of two production periods. In the first period firms can either produce some quantity or decide to wait. If, and only if, a firm decides to wait it is informed about the opponent's first period action and after that can choose it's second-period production. Hamilton and Slutsky show that this game has three subgame perfect Nash equilibria. One simultaneous-move Cournot equilibrium where both firms produce the Cournot-Nash quantities in the first production period.[17] Two sequential-move Stackelberg equilibria where one firm produces the Stackelberg leader's quantity in the first production period and the other firm produces the Stackelberg follower's quantity in the second production period.[18] Thus, the set of equilibria in Hamilton and Slutsky's game is given by

$$E_{HS} = \left\{ \left(q_1^1, q_1^2\right) = (N, N) \right\} \cup \left\{ \left(q_1^1, q_2^2\right) = (S, R(S)) \right\} \cup \left\{ \left(q_2^1, q_1^2\right) = (R(S), S) \right\}.$$

The Stackelberg equilibria are in undominated strategies. The simultaneous-move equilibrium uses weakly dominated strategies since playing the Cournot-Nash quantity in the first production period is dominated by waiting to play after one's rival.

4 Inequity Aversion

Many experiments indicate that individuals are not only motivated by material self-interest, but also care about the well-being of others. We incorporate this possibility in Saloner's game by assuming that firms are averse to inequality in profits. To model this, we make use of Fehr and Schmidt's (1999) approach. Thus, we assume that firm i's payoff is given by

$$U^i(\pi^i, \pi^j) = \pi^i - \left[\alpha_i \max\left(\pi^j - \pi^i, 0\right) + \beta_i \max\left(\pi^i - \pi^j, 0\right)\right], \ i \neq j = 1, 2.$$

The terms in the square bracket are the payoff effects of disadvantageous and advantageous inequity, respectively. When $\pi^j > \pi^i$ firm i feels envy of firm j, this is the disadvantageous inequity term. When $\pi^j < \pi^i$ firm i feels compassion for firm j, this is the advantageous inequity term. Fehr and Schmidt assume that α_i and β_i are nonnegative, that $\alpha_i > \beta_i$, that is, the dislike of disadvantageous

[17] Both firms producing the Cournot-Nash quantities in the second production period is not an equilibrium since each firm would do better by unilaterally deviate and produce the Stackelberg leader's quantity in the first production period.

[18] A firm producing the Stackelberg leader's quantity, S^i, in the first production period and the opponent producing the Stackelberg follower's quantity, $R^j(S^i)$, in the first production period is not an equilibrium because the leader would rather produce it's best response to the Stackelberg follower's quantity, that is $R^i(R^j(S^i))$.

inequity is stronger than that of advantageous inequity, and that β_i is smaller than 1. We assume that α_i is nonnegative and that $\beta_i \in [0, 1/2]$.[19]

Santos-Pinto (2006) shows that the single-period reaction function of firm i, $i \neq j = 1, 2$, in the presence of inequity aversion is defined by

$$R^i(q^j) = \begin{cases} s^i(q^j), & 0 \leq q^j \leq q(\beta_i) \\ q^j, & q(\beta_i) \leq q^j \leq q(\alpha_i) \\ t^i(q^j), & q(\alpha_i) \leq q^j \end{cases},$$

where

$$s^i(q^j) = \arg_{q^i} \max (1 - \beta_i) \left[P\left(q^i + q^j\right) - c_i \right] q^i + \beta_i \left[P\left(q^i + q^j\right) - c_j \right] q^j, \quad (2)$$

$$t^i(q^j) = \arg_{q^i} \max (1 + \alpha_i) \left[P\left(q^i + q^j\right) - c_i \right] q^i - \alpha_i \left[P\left(q^i + q^j\right) - c_j \right] q^j, \quad (3)$$

$q(\beta_i)$ is the solution to

$$(1 - \beta_i) \left[P(2q) - c_i \right] + P'(2q)q = 0, \quad (4)$$

and $q(\alpha_i)$ is the solution to

$$(1 + \alpha_i) \left[P(2q) - c_i \right] + P'(2q)q = 0. \quad (5)$$

The main difference between these reaction function and the standard reaction functions is that with inequity aversion there is a range of an opponent's output levels for which the best response of a firm is to produce the same quantity as the opponent. That happens around the Cournot-Nash equilibrium quantity of the standard simultaneous-move game. In other words, the best response has a positive slope for output levels of the opponent close to the Cournot-Nash level and a negative slope for the remaining output levels of the opponent. As we have seen, Huck et al.'s (2002) experiment on Hamilton and Slutsky's action commitment game find evidence for this type of reaction function.

Santos-Pinto (2006) also shows that the set of Nash equilibria of the single-period symmetric Cournot duopoly game when firms are averse to inequity is given by

$$E^{IA} = \left\{ \left(q^1, q^2\right) : q^1 = q^2, \text{ and } N(\beta_1, \beta_2) \leq q^i \leq N(\alpha_1, \alpha_2),\ i = 1, 2 \right\},$$

where $N(\beta_1, \beta_2) = \max\left[q\left(\beta_1\right), q(\beta_2)\right]$, and $N(\alpha_1, \alpha_2) = \min\left[q(\alpha_1), q(\alpha_2)\right]$.

This result tells us that inequity aversion between firms gives rise to a continuum of symmetric equilibria in the single-period Cournot duopoly game. The intuition for this result is as follows. Suppose that a firm knows it's opponent will produce an output level that is close to the Nash equilibrium of the standard single-period Cournot game. If that firm dislikes inequity aversion, then

[19] The assumption that β_i is smaller than $1/2$ implies that a firm never cares more about the profit of her opponent than about her own profit. This assumption also rules out equilibria of the single period Cournot model where firms produce less than the collusive quantities.

there is a cost in advantageous inequity associated with producing a higher level of output than the opponent. Similarly, there is also a cost in disadvantageous inequity associated with producing a smaller output level than the opponent. For a range of output levels close to the Nash equilibrium of the standard single-period Cournot game the profits lost from not matching the opponent's output are small while the inequity costs are large. If that is the case then the firm is better off by producing the same level of output as the opponent.

The result also shows that the smallest Nash equilibria of the single-period Cournot game is determined by the lowest level of compassion of the two firms and that the largest Nash equilibria is determined by the lowest level of envy of the two firms. We see from (2) that if both firms have a level of compassion equal to 1/2, then the lowest Nash equilibrium of the single-period Cournot duopoly game with inequity averse firms corresponds to the collusive outcome.

We will now show that inequity aversion between firms also gives rise to a continuum of symmetric equilibria in Saloner's two-period Cournot duopoly game. We start our analysis by stating a lemma that characterizes a firm's second-period equilibrium output in Saloner's game with inequity averse firms. Saloner (1987) proved this lemma in the standard two-period Cournot duopoly game with selfish firms. To prove our lemma we assume, without loss of generality, that there is symmetry in the inequity aversion parameters, that is, we take $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$.[20] Given this assumption, we let $N(\beta)$ denote $N(\beta_1, \beta_2)$ and $N(\alpha)$ denote $N(\alpha_1, \alpha_2)$. We are now ready to state the lemma.

Lemma 1 *Given the first-period outputs (q_1^1, q_1^2), the second-period equilibrium outputs for firm i, $i = 1, 2$, are as follows:*

$$q_2^i = \begin{cases} 0 & \text{if } q_1^i \geq t^i(q_1^j), \text{ and } q_1^j \geq t^j(q_1^i) \\ N(\beta) - q_1^i & \text{if } q_1^i \leq N(\beta), \text{ and } q_1^j \leq N(\beta) \\ q_1^j - q_1^i & \text{if } q_1^i \leq q_1^j, \text{ and } N(\beta) \leq q_1^j \leq N(\alpha) \\ 0 & \text{if } N(\beta) \leq q_1^i \leq N(\alpha), \text{ and } q_1^j \leq q_1^i \\ 0 & \text{if } q_1^i \geq N(\alpha), \text{ and } q_1^j \leq t^j(q_1^i) \\ t^i(q_1^j) - q_1^i & \text{if } q_1^i \leq N(\alpha),\ N(\alpha) \leq q_1^j, \text{ and } q_1^i \leq t^i(q_1^j) \end{cases}, \tag{6}$$

where t^i is given by (3).

This lemma says if $\left(q_1^1, q_1^2\right)$ lies on or outside the outer-envelope of $t^1(q^2)$ and $t^2(q^1)$, then neither firm produces in the second period. If $\left(q_1^1, q_1^2\right)$ lies inside the outer-envelope of $t^1(q^2)$ and $t^2(q^1)$, then there are three cases to be considered. If both firms have produced less than $N(\beta)$ in the first period, then each produces up to $N(\beta)$ in the second period. If firm j has produced q_1^j, where q_1^j is more than $N(\beta)$ but less than $N(\alpha)$, and firm i has produced less than q_1^j, then firm i produces it's best response to firm j's first period output and firm j does not produce at all. In this case, both firms end up producing the same

[20] If we assume that $\beta_1 \neq \beta_2$ and/or $\alpha_1 \neq \alpha_2$ the game becomes asymmetric and firms' second period equilibrium outputs are slightly more complicated to obtain. However, this has no implications on the analysis that follows.

amount since inequity aversion implies that firm i's best response to firm j's first period output is to produce as much as firm j. If one firm has exceeded $N(\alpha)$ but the other has not, then the latter produces it's best response to the former's first period output and the former does not produce at all. In this case, the firm that has exceeded $N(\alpha)$ produces more than the firm that has not exceeded $N(\alpha)$.

By comparing (1) to (6) we can see that inequity aversion between firms changes the second-period equilibrium outputs in two ways. First, there is a range of first-period output levels for which the best response of a firm is to produce the same level of output as the opponent. This happens because producing the same level of output as the opponent implies a first order gain in reduction of inequity costs and a second order loss in material payoff. Second, there is another range of first period outputs for which the best response of a firm is shifted by comparison with the best response in the absence of inequity aversion. This happens because, for this other range of first period output levels, the firm that produces the smallest output level in the first production end ups feeling envy towards the opponent. This implies that the second-period output of the firm that feels envy is larger than the output it would have produced in the second period if it felt no envy.

We will use lemma 1 to characterize the set of equilibria of Saloner's game with inequity averse firms. Before doing that we need to introduce some notation. Let the Stackelberg leader's quantity in the presence of inequity aversion be denoted by $S^i(\alpha, \beta)$, $i = 1, 2$ and the Stackelberg follower's quantity by $R^j(S^i(\alpha, \beta))$, $j \neq i$. If firm i is the Stackelberg leader, then it picks the point in $R^j(q^i)$ that maximizes its payoff. The existence of inequity aversion implies that the Stackelberg leader's quantity is defined as

$$S^i(\alpha,\beta) = \begin{cases} N(\beta), & \text{if } U^i(L^i(\alpha,\beta), t^j(L^i(\alpha,\beta))) \leq U^i(N(\beta), N(\beta)) \\ L^i(\alpha,\beta), & \text{otherwise} \end{cases}, \tag{7}$$

and the Stackelberg follower's quantity by

$$R^j\left(S^i(\alpha,\beta)\right) = \begin{cases} N(\beta), & \text{if } U^i(L^i(\alpha,\beta), t^j(L^i(\alpha,\beta))) \leq U^i(N(\beta), N(\beta)) \\ t^j\left(L^i(\alpha,\beta)\right), & \text{otherwise} \end{cases}, \tag{8}$$

where

$$\begin{aligned} L^i(\alpha,\beta) = \arg_{q^i \geq N(\alpha)} \max\, (1-\beta)\left[P\left(q^i + t^j(q^i)\right) - c_i\right] q^i \\ + \beta\left[P\left(q^i + t^j(q^i)\right) - c_j\right] t^j(q^i), \end{aligned}$$

$j \neq i = 1, 2$. We see from (7) and (8) that the presence of inequity aversion implies that the Stackelberg point is either point $(N(\beta), N(\beta))$, the smallest Nash equilibrium of the simultaneous-move game, or point $\left(L^i(\alpha,\beta), t^j\left(L^i(\alpha,\beta)\right)\right)$. If the payoff of the smallest Nash equilibrium of the simultaneous move game is greater than the payoff of point $\left(L^i(\alpha,\beta), t^j\left(L^i(\alpha,\beta)\right)\right)$, then point $(N(\beta), N(\beta))$

is the Stackelberg point. If the payoff of the smallest Nash equilibrium of the simultaneous move game is smaller than the payoff of point $\left(L^i(\alpha,\beta), t^j\left(L^i(\alpha,\beta)\right)\right)$, then point $\left(L^i(\alpha,\beta), t^j\left(L^i(\alpha,\beta)\right)\right)$ is the Stackelberg point. In this case, firm i produces more than firm j since $L^i(\alpha,\beta) < t^j\left(L^i(\alpha,\beta)\right)$. This implies that the material payoff of firm i is larger than the material payoff of firm j and therefore firm i feels compassion towards firm j whereas firm j feels envy towards firm i. We also see from (7) and (8) that if the Stackelberg point is point $\left(L^i(\alpha,\beta), t^j\left(L^i(\alpha,\beta)\right)\right)$, then it a function of α and of β. An increase in envy reduces $L^i(\alpha,\beta)$ and so does an increase in compassion. If the degree of envy increases, this leads the follower to raise production and this in turn implies a lower quantity for the leader. Also, if the degree of compassion of the leader increases, then the leader reduces its output to reduce inequity aversion.

Proposition 1 characterizes the set of equilibria of Saloner's game for relatively high levels of inequity aversion between firms.

Proposition 1 *If $U^i(N(\beta),N(\beta)) > U^i(S^i(\alpha,\beta), t^j(S^i(\alpha,\beta)))$, $i=1,2$, then the set of equilibria of Saloner's game with inequity averse firms is given by*

$$E_S^{IA} = \left\{\left(q^1,q^2\right) : q^1 = q^2, \textit{and } N(\beta) \leq q^i \leq N(\alpha),\ i=1,2\right\}.$$

This result tells us that if the degree of inequity aversion between firms is relatively high, then Saloner's game has a continuum of symmetric SPNE. The set of equilibria is any pair of total outputs where firms produce the same quantity, that is $q_1^1+q_2^1 = q_1^2+q_2^2$, and where the quantities produced by the two firms are between the smallest and the largest Nash equilibrium of the single-period Cournot duopoly game with inequity averse firms. Thus, if the degree of inequity aversion between firms is relatively high, then the set of SPNE of Saloner's game coincides with the set of Nash equilibria of the single-period Cournot duopoly game.

The intuition for this result is straightforward. Inequity aversion between firms, no matter if it is high or low, gives rise to a continuum of symmetric equilibria both in the single-period Cournot game as well as in Saloner's duopoly game. Inequity aversion between firms makes symmetric outcomes in the set E_S^{IA} more desirable to firms than unilateral deviations to asymmetric outcomes. This happens because any unilateral deviation away from a symmetric outcome in E_S^{IA} with $N \leq q^i \leq N(\alpha)$, $i=1,2$, leads to an inequity cost and a loss in material payoff and any unilateral deviation from a symmetric outcome in E_S^{IA} with $N(\beta) \leq q^i \leq N$, $i=1,2$, leads to a first order inequity cost and a second order gain in material payoff.

Relatively high levels of inequity aversion rule out asymmetric equilibria. If α and β are such that the leader's payoff in the asymmetric Stackelberg point is strictly smaller than the payoff in the smallest Nash equilibrium of the single-period Cournot duopoly game, then the asymmetric Stackelberg point is not an equilibrium of Saloner's game. If that is the case, then there is no other asymmetric equilibria since the payoff of the firm that produces more in any asymmetric outcome is always smaller than its Stackelberg leader's payoff. In short, relatively high levels of inequity aversion increase the inequity costs of choosing a first period production that leads to asymmetric quantities and so the game has no asymmetric equilibria.[21]

Our next result characterizes the set of equilibria of Saloner's game for relatively low levels of inequity aversion between firms.

Proposition 2 *If α and β are such that the Stackelberg point exists and $U^i(S^i(\alpha,\beta), R^j(S^i(\alpha,\beta))) > U^i(N(\beta), N(\beta))$, $i = 1, 2$, then the set of equilibria of Saloner's game with inequity averse firms is given by*

$$\begin{aligned} E_S^{IA} &= \left\{ \left(q^1, q^2\right) : q^1 = q^2, \text{and } N(\beta) \le q^i \le N(\alpha),\ i = 1, 2 \right\} \\ &\cup \left\{ \left(q^1, q^2\right) : t^2(L^1(\alpha,\beta)) \le q^2 \le \hat{q}(\alpha,\beta),\ q^1 = \left(t^2\right)^{-1}\left(q^2\right) \right\} \\ &\cup \left\{ \left(q^1, q^2\right) : t^1(L^2(\alpha,\beta)) \le q^1 \le \hat{q}(\alpha,\beta),\ q^2 = \left(t^2\right)^{-1}\left(q^1\right) \right\}, \end{aligned}$$

where $\hat{q}(\alpha,\beta)$ is the solution to $U^i(N(\beta), N(\beta)) = U^i(q^i, t^j(q^i))$.

This result tells us that if the degree of inequity aversion between firms is relatively low, then Saloner's game has a continuum of symmetric and asymmetric SPNE. Like in Proposition 1, the set of equilibria is any pair of total outputs where firms produce the same quantity and where the quantities produced by the two firms are between the smallest and the largest Nash equilibrium of simultaneous move game. Additionally, the set of equilibria is also composed of any point on the outer envelope of the reaction function of firm i between firm i's Stackelberg leader's output and output level $\hat{q}(\alpha,\beta)$, $i = 1, 2$.

This result is not surprising. If the level of inequity aversion is relatively low, then unilateral deviations from asymmetric outcomes to symmetric outcomes may no longer be desirable. This happens because as inequity aversion decreases the gains in inequity costs from playing symmetric outcomes become smaller than the losses in material payoffs. If that is the case there must be sufficiently low levels of inequity aversion for which Saloner's game has asymmetric equilibria. Proposition 2 states the conditions for this to happen and characterizes the set of asymmetric equilibria. It tells us that if α and β are relatively low, then

[21] To characterize the condition that defines what we call relatively high levels of inequity aversion consider a linear demand: $P = a - bQ$. In this case we have that $N(\beta) = \frac{1-\beta}{3-2\beta}\frac{a-c}{b}$ and $S^i(\alpha,\beta) = \frac{(1-2\beta)(1+\alpha)^2}{(1+2\alpha)(2+2\alpha-3\beta-2\alpha\beta)}\frac{a-c}{b}$. The condition $U^i(N(\beta), N(\beta)) \ge U^i(S^i(\alpha,\beta), t^j(S^i(\alpha,\beta)))$ becomes

$$\frac{(1-\beta)}{(3-2\beta)^2}\frac{(a-c)^2}{b} \ge \frac{(1+\alpha-\beta)^2}{4(1+2\alpha)(2+2\alpha-3\beta-2\alpha\beta)}\frac{(a-c)^2}{b},$$

or

$$4(1-\beta)(1+2\alpha)(2+2\alpha-3\beta-2\beta\alpha) \ge (3-2\beta)^2(1+\alpha-\beta)^2.$$

Solving the condition as an equality with respect to α we have

$$\alpha = \frac{1}{2(6\beta-7)}\left(-4\beta^2 - 2\beta + 6 - 4\sqrt{\left(4\beta^4 - 16\beta^3 + 25\beta^2 - 17\beta + 4\right)}\right).$$

We see that if $\beta = 0$, then the minimum degree of envy that satisfies the inequality is $\alpha = 1/7$. If $\alpha = 0$, then the minimum degree of compassion that satisfies the inequality is $\beta = 0.14922$.

the Stackelberg point is given by $\left(L^i(\alpha,\beta), t^j\left(L^i(\alpha,\beta)\right)\right)$. This point is an equilibrium of the two-period game since the condition $U^i(S^i(\alpha,\beta), t^j(S^i(\alpha,\beta))) \geq U^i(N(\beta), N(\beta))$ implies that the Stackelberg leader does not wish to deviate from the Stackelberg quantity to $N(\beta)$, the best symmetric equilibrium quantity. It also follows that there is a set of points on $t^j(\cdot)$ such that $\hat{q}(\alpha,\beta) < q^i < L^i(\alpha,\beta)$ which are also asymmetric equilibria since the payoff at these points is larger than the payoff of the smallest Nash equilibrium of the simultaneous move game.

Santos-Pinto (2006) shows that the point $(N(\beta), N(\beta))$ is decreasing with β, that is, the smallest symmetric equilibrium of the single-period Cournot duopoly game with inequity averse firms is decreasing with an increase in compassion. This means, that an decrease in compassion moves the set of symmetric equilibrium outcomes closer to the collusive outcome (the outcome obtained when $\beta = 1/2$). By contrast, the largest symmetric equilibrium of the single-period Cournot duopoly game with inequity averse firms is increasing with an increase in envy.

Obviously, these results also apply to the set of symmetric equilibria of Saloner's game with inequity averse firms. As α and β converge to zero the impact of inequity aversion vanishes. As the impact of inequity aversion vanishes the set of symmetric equilibria in Saloner's game with inequity aversion collapses to the Nash equilibria of the single-period Cournot game.[22] By contrast, as the impact of inequity aversion vanishes the set of asymmetric equilibria in Saloner's game with inequity aversion expands.[23]

Let us now consider the impact of inequity aversion on the set of equilibria in Hamilton and Slutsky's action commitment game. The next result characterizes the set of equilibria of this game for relatively high levels of inequity aversion.

Proposition 3 *If* $U^i(N(\beta), N(\beta)) > U^i(S^i(\alpha,\beta), t^j(S^i(\alpha,\beta)))$, $i = 1, 2$, *then the set of equilibria of Hamilton and Slutsky's action commitment game with inequity averse firms is given by*

$$E_{HS}^{IA} = \left\{\left(q_1^1, q_1^2\right) : q_1^1 = q_1^2, \textit{and } N(\beta) \leq q_1^i \leq N(\alpha),\ i = 1, 2\right\} \\ \cup \left\{\left(q_2^1, q_2^2\right) = (N(\beta), N(\beta))\right\}.$$

This result tells us that if the degree of inequity aversion between firms is relatively high, then Hamilton and Slutsky's action commitment game has a continuum of symmetric SPNE. The set of equilibria is any pair of outputs where firms produce the same quantity, they do it in the first production period, and where the quantities produced by the two firms are between the smallest and the largest Nash equilibrium of the single-period Cournot duopoly game with inequity averse firms. Thus, if the degree of inequity aversion between firms is relatively high, then the set of SPNE of Hamilton and Slutsky's action

[22] As α converges to zero the point $(N(\alpha), N(\alpha))$ converges to (N, N) and as β converges to zero the point $(N(\beta), N(\beta))$ converges to (N, N).

[23] As α and β converge to zero the point $\left(S^i(\alpha,\beta), t^j(S^i(\alpha,\beta))\right)$ converges to $\left(S^i; R^j(S^i)\right)$, $i = 1, 2$..As α and β converge to zero the point $\left(\hat{q}(\alpha,\beta), t^j(\hat{q}(\alpha,\beta))\right)$ converges to (N, N).

commitment game coincides with the set of Nash equilibria of the single-period Cournot duopoly game.[24]

The intuition for this result is as follows. Inequity aversion between firms, no matter if it is high or low, gives rise to a continuum of symmetric equilibria both in the single-period Cournot game as well as in Hamilton and Slutsky's game. Additionally, if inequity aversion is relatively high, that is, α and β are such that each firm prefers the smallest Nash equilibrium payoff of the simultaneous move game, $U^i(N(\beta), N(\beta))$ to its payoff as the Stackelberg leader, then there are no Stackelberg equilibria. Thus, the only equilibria of Hamilton and Slutsky's action commitment game with relatively high levels of inequity aversion between firms are the simultaneous-move equilibria. The fact that in Hamilton and Slutsky's action commitment game firms can only produce in one of the two periods implies that production in any simultaneous move-equilibria takes place in the first period.

The fact that there exists a continuum of symmetric equilibria and that firms must coordinate by moving simultaneously in the first production period is consistent with the empirical finding that there is more coordination failure in Hamilton and Slutsky's action commitment game than in Saloner's game.

The next result characterizes the set of equilibria in Hamilton and Slutsky's action commitment game for relatively low levels of inequity aversion.

Proposition 4 *If α and β are such that the Stackelberg point exists and $U^i(S^i(\alpha,\beta), R^j(S^i(\alpha,\beta))) > U^i(N(\beta), N(\beta))$, $i = 1,2$, then the set of equilibria of Hamilton and Slutsky's action commitment game with inequity averse firms is given by*

$$E_{HS}^{IA} = \left\{\left(q_1^1, q_1^2\right) : q_1^1 = q_1^2, \text{ and } N(\beta) \leq q_1^i \leq N(\alpha),\ i = 1,2\right\}$$
$$\cup\left\{\left(q_1^1, q_2^2\right) = (L^1(\alpha,\beta), t^2(L^1(\alpha,\beta)))\right\}\cup\left\{\left(q_2^1, q_1^2\right) = (t^1(L^2(\alpha,\beta)), L^2(\alpha,\beta))\right\}.$$

This result tells us that if the degree of inequity aversion between firms is relatively low, then Hamilton and Slutsky's action commitment game has a continuum of symmetric SPNE and two asymmetric SPNE. In any symmetric equilibria both firms produce in the first period and each firm produces a quantity between the smallest and the largest Nash equilibrium quantity of the single-period Cournot duopoly game with inequity averse firms. The asymmetric equilibria are of the leader follower type with one firm producing the Stackelberg leader's quantity in the first production period and the other firm producing the Stackelberg follower's quantity in the second period. The difference here, by comparison with Hamilton and Slutsky's action commitment game with selfish firms, is that a compassionate leader produces less than a selfish leader and a envious follower produces more than a selfish follower.

[24] This is true for any symmetric equilibria in E_{HS}^{IA}, except the lowest Nash equilibrium of the simultanous-move Cournot game, $(N(\beta), N(\beta))$. Suppose that both firms produce $N(\beta)$ in the second production period. In this case ,each firm is indifferent between producing $N(\beta)$ in the second production period or in the first.

5 Summary and Comparison

In this section we summarize the predictions of the inequity aversion explanation for Saloner's game and for Hamilton and Slutsky's action commitment game. We also compare the predictions to the experimental evidence. Table V below summarizes the predictions for Saloner's game.

Table V

Saloner's Duopoly Game							
	Sym. eq.	Asym eq.	Coll. out.	Stack. warf.	Time prod.	Balance shares	Prod. P1
Ineq. Av.							
High	Many	-	Yes	Yes	P1&P2	Yes	No
Low	Many	-	No	No	P1&P2	Yes	No
	-	Many	-	-	P1&P2	-	No

Recall that the experimental evidence on Saloner's game tells us that: (i) Stackelberg outcomes are extremely rare, (ii) simultaneous-move symmetric outcomes are the most frequent outcomes, (iii) sometimes collusive outcomes are observed, (iv) there is production in both periods with 84% of production taking place in the first period, (v) subjects seem to attempt to balance market shares in the second production period, and (vi) subjects do not produce more than the Stackelberg follower's quantity in the first production period.

Table V shows us that the predictions of the inequity aversion explanation are consistent with most of the experimental evidence on Saloner's game. First, relatively high levels of inequity aversion imply that Saloner's game has a continuum of simultaneous-move symmetric equilibria. Among all the symmetric equilibria the Cournot-Nash equilibrium may be the one that is more frequently played because it is always a subgame perfect Nash equilibrium of the game no matter if there is inequity aversion or not. This is no longer true for other symmetric equilibria. When inequity aversion is low there is still a continuum of simultaneous-move symmetric equilibria but there is also a continuum of asymmetric equilibria where play may be sequential.

Second, inequity aversion is also able to explain the fact that in some games "collusive outcomes" are played. This can happen whenever two subjects with a high degree of compassion are matched to play the game and are able to coordinate on the collusive outcome. Similarly, inequity aversion is also able to explain the fact that in some games there is Stackelberg warfare. This can happen whenever two subjects with a high degree of envy are matched to play the game and both produce more than the Cournot-Nash quantities.

Third, relatively high levels of inequity aversion rule out Stackelberg outcomes. However, relatively low levels of inequity aversion do not. Thus, whenever two subjects with a relatively low level of inequity aversion are matched to play the game we may have Stackelberg equilibria.

Fourth, inequity aversion is also able to explain the fact that subjects in experiments produce in both periods. This happens because inequity aversion gives rise to a multiplicity of symmetric equilibria in the game and subjects have to coordinate on one of them. If subjects are unable to coordinate in one of the multiple symmetric equilibria in the first production period, then they have an incentive to produce in the second production period to attain coordination before the market clears.

Fifth, inequity aversion also predicts that subjects attempt to balance market shares. This happens whenever subjects fail to coordinate on a symmetric equilibrium in the first production period. In those cases, the subject who produced less in the first production period will produce a positive amount in the second period whereas the subject who produced more in the first production period will not produce in the second production period.

The only empirical finding in Saloner's game that the inequity aversion explanation seems unable to account for is the fact that firms do not produce more than the Stackelberg follower's quantity in the first production period.

Table VI below summarizes the predictions for Hamilton and Slutsky's action commitment game.

Table VI

Hamilton and Slutsky's Action Commitment Game							
	Sym. eq.	Stack. eq.	Coll. out.	Stack. warf.	Punish leader	Time prod.	Cournot in P2
Ineq. Av.							
High	Many	-	Yes	Yes	-	P1	No
Low	Many	-	No	No	-	P1	No
	-	Two	-	-	Yes	P1&P2	-

Recall that the experimental evidence on Hamilton and Slutsky's action commitment game tells us that: (i) Stackelberg outcomes are rare, (ii) simultaneous-move Cournot outcomes are the most frequent outcomes, (iii) simultaneous-move outcomes are often played in the second production period, and (iv) behavior is quite heterogeneous–in some cases followers punish leaders, in other cases collusive outcomes are played, and in other cases Stackelberg warfare is observed.

Table VI shows us that inequity aversion is also able to explain most of the experimental evidence on Hamilton and Slutsky's action commitment game. First, relatively high levels of inequity aversion imply that Hamilton and Slutsky's action commitment game only has simultaneous-move symmetric outcomes where both firms produce in the first production period.[25] When inequity aversion is low there is a continuum of simultaneous-move symmetric equilibria but there are also two Stackelberg equilibria with sequential play.

[25] Like in Saloner's game with inequity averse firms, among all the symmetric equilibria in Hamilton and Slutsky's action commitment game with inequity averse firms, the Cournot-Nash equilibrium may be the one that is more fequently played. This happens because this equilibrium is always a subgame perfect Nash equilibrium of the game no matter if there is inequity aversion or not. That is no longer the case with other symmetric equilibria.

Second, inequity aversion can explain collusive outcomes in Hamilton and Slutsky's action commitment game. This happens whenever both players have a relatively high level of inequity aversion and they are able to coordinate on the collusive outcome.

Third, if inequity aversion is relatively high there are no Stackelberg outcomes in Hamilton and Slutsky's action commitment game. So, for Stackelberg outcomes to be played players must have relatively low levels of inequity aversion.

Fourth, if inequity aversion is relatively low and players play the Stackelberg outcome, then the model predicts that the Stackelberg leader will feel compassion towards the follower and that the Stackelberg follower will feel envy towards the leader. This implies that a compassionate leader produces less than a selfish leader and that an envious follower produces more than a selfish follower. This pattern is consistent with the evidence in Huck et al. (2002). Table III shows that in the experiment with the large payoff matrix, explicit followers produce on average 8.93 units. This is significantly higher than the Stackelberg follower's quantity of 6 units.[26]

The only empirical finding in Hamilton and Slutsky's action commitment game that inequity aversion is unable to explain is simultaneous-move Cournot-Nash outcomes in the second production period.[27]

6 Extensions

As we have seen, Fehr and Schmidt's (1999) model of inequity aversion is able to explain several experimental findings in endogenous timing games. However, Fehr and Schmidt's specification is a particular functional form of inequity aversion (it is piecewise linear and non-differentiable). Could it be that the results obtained extend to more general preferences?

[26] The same happens in the experiment with the small payoff matrix. On average, explicit followers in the experiment with the small payoff matrix produce 7.89. Huck et al. (2002) do not display data for explicit leaders. However, we can use the data in the small payoff matrix to have an idea of the average quantity of explicit leaders (in the small payoff matrix most players who produce in the first period are explicit leaders, this is not the case in the large payoff matrix). In the experiment with the small payoff matrix there are 136 players who produce in the first period, of which 94 are explicit leaders and 42 are players who produce simultaneously. If the 94 explicit leaders produced the leader's quantity, 12 units, and the other 42 players the Cournot-Nash quantity, the average output of these 136 players should be equal to 10.76. By contrast, the data shows that the average output of these 136 players is significantly lower: 8.65 units. This tells us that, on average, explicit leaders produce substantially less than the Stackelberg quantity.

[27] Fonseca et al. (2005b) test experimentally Hamilton and Slutsky (1990)' s observable delay game. In this game two firms bindingly announce a production period (one out of two periods) and then produce in the announced sequence. Hamilton and Slutsky show that this game has a unique symmetric equilibrium where firms produce only in the first period. Fonseca et al. (2005b) find that there is delay in players' production decisions. The findings in this paper show that inequity aversion is also not able to explain delay in Fonseca et al. (2005b).

Santos-Pinto (2006) studies the impact of general forms of inequity aversion on Cournot competition. He shows that for differentiable forms of inequity aversion the best reply of a firm is always negatively sloped. However, the best reply of an inequity averse firm is smaller than the best reply of a selfish firm when the rival produces low output levels (the inequity averse firms fells compassion for the rival) and the best reply of an inequity averse firm is larger than that of a selfish firm when the rival produces high output levels (the inequity averse firm feels envy towards the rival). This implies that the set of SPNE of Saloner's game with differentiable inequity aversion is closer to the 45^odegree line, than the set of SPNE of Saloner's game with selfish firms. The same happens in Hamilton and Slutsky's (1990) endogenous timing game. The Stackelberg equilibria of Hamilton and Slutsky's action commitment game with firms with differentiable inequity aversion are much less asymmetric than the Stackelberg equilibria obtained with selfish firms. Thus, inequity aversion either rules out asymmetric outcomes completely (high levels of piecewise linear inequity aversion) or reduces the degree of asymmetry substantially (high levels of differentiable inequity aversion).

The fact that differentiable inequity aversion does not lead to positively sloped best replies for intermediate output levels of the rival implies that the continuum of equilibria result obtained with piecewise linear aversion is no longer valid. This in turn implies that differentiable inequity aversion can no longer explain production in both periods in Saloner's game as well as the finding that players try to balance market shares in the second production period.

Besides inequity aversion, reciprocity, is another common type of other-regarding preferences. A reciprocal agent cares about the intentions of his rivals. He responds to actions that he perceives to be harmful in a harmful manner and he responds to actions that he perceives to be kind in a kind manner. Santos-Pinto (2006) shows that the impact of reciprocity on Cournot competition is similar to that of inequity aversion. Thus, the results in this paper also extend to players with reciprocal preferences.

7 Conclusion

The prediction of asymmetric equilibria with Stackelberg outcomes is clearly the most frequent result in the endogenous timing literature. Several experiments have tried to validate this prediction empirically, but failed to find support for it. By contrast, the experiments find that simultaneous-move outcomes are modal. Additionally, the experiments find that behavior in endogenous timing games is quite heterogeneous. In many cases we have simultaneous-move play, sometimes collusive outcomes are observed, other times there is Stackelberg warfare, and there can be also sequential play with Stackelberg like outcomes, among others.

The gap between the theory and the experimental evidence was the main motivation behind this paper. To bridge that gap the paper formalizes the implications of inequity aversion in Saloner's and Hamilton and Slutsky's endogenous timing games. The paper shows that inequity aversion is able to organize most of the experimental evidence on endogenous timing games.

We find that relatively high levels of inequity aversion rule out asymmetric equilibria in endogenous timing games. We also find that inequity aversion gives rise to a continuum of simultaneous-move equilibria. These equilibria include the Cournot-Nash outcome, collusive outcomes as well as Stackelberg warfare. Thus, inequity aversion is able to explain the wide diversity of behavior observed in experimental endogenous timing games. Inequity aversion is also able to explain why subjects produce in both production periods in Saloner's game. This happens whenever individuals fail to coordinate on one of the symmetric equilibria in the first production period. However, inequity aversion is not able to explain delay in Hamilton and Slutsky's endogenous timing games. Inequity aversion is also not able to explain the fact that firms do not produce more than the Stackelberg follower's output in the first production period in Saloner's game.

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8 Appendix

Proof of Proposition 1: We first show that any point in E_S^{IA} can be attained as a subgame perfect Nash equilibrium. To do that we will show that the following strategies for each firm constitute a subgame perfect Nash equilibrium: $q_1^i = a^i$, where $\left(a^1, a^2\right)$ is a point in E_S^{IA} and q_2^i is given by (6). Lemma 1 shows that the strategies in the second production period are equilibrium strategies. Let us then consider the first production period. Since the game is symmetric we can focus on a single firm. So, without loss of generality, consider firm 1. If firm 1 follows the prescribed strategy it's payoff is given by $U^1(a^1, a^2) = \pi^1(a^1, a^2)$. It is obvious that firm 1 can not gain by producing less than it's prescribed first period strategy. Suppose that firm 1 deviates from it's first period strategy and produces $q_1^1 = a^1 + \varepsilon$, with $0 < \varepsilon \leq N(\alpha) - a^1$. If that is the case, then firm 1 will not produce in the second period but firm 2 will produce $q_2^2 = \varepsilon$. So, the total output of each firm becomes $q^i = a^i + \varepsilon$, $i = 1, 2$. This implies that firm 1's payoff under the deviation is given by $\pi^1(a^1 + \varepsilon, a^2 + \varepsilon)$. If $\beta_i \leq 1/2$, $i = 1, 2$, and $0 < \varepsilon \leq N(\alpha) - a^1$, then $\pi^1(a^1 + \varepsilon, a^2 + \varepsilon) < \pi^1(a^1, a^2)$ since firms are moving away from the collusive quantities. Thus, firm 1 is worse off by producing $q_1^1 = a^1 + \varepsilon$, with $0 < \varepsilon \leq N(\alpha) - a^1$, rather that producing what it's prescribed first period strategy dictates. Now, suppose that firm 1 deviates from it's first period strategy and produces $q_1^1 = a^1 + \varepsilon$, with $N(\alpha) - a^1 < \varepsilon \leq (t^2)^{-1}(a^2) - a^1$. If that is the case, then firm 1 will not produce in the second period and firm 2 will produce $q_2^2 = t^2(q_1^1) - a^1$. Thus, we have that $q^1 = a^1 + \varepsilon$ and $q^2 = t^2(a^1 + \varepsilon)$, with $N(\alpha) - a^1 < \varepsilon \leq (t^2)^{-1}(a^2) - a^1$. The payoff of firm 1 becomes $U^1\left(a^1 + \varepsilon, t^2(a^1 + \varepsilon)\right)$. We know that

$$U^1\left(a^1 + \varepsilon, t^2(a^1 + \varepsilon)\right) < U^1\left(t^2(a^1 + \varepsilon), t^2(a^1 + \varepsilon)\right).$$

We also know that

$$U^1\left(t^2(a^1 + \varepsilon), t^2(a^1 + \varepsilon)\right) < U^1(a^1, a^2).$$

These two inequalities imply that

$$U^1\left(a^1 + \varepsilon, t^2(a^1 + \varepsilon)\right) < U^1(a^1, a^2),$$

that is, firm 1 can not gain with the deviation.

It remains to show that there is no other subgame perfect Nash equilibrium. Suppose there is another equilibrium $\left(b^1, b^2\right) \notin E_S^{IA}$. Clearly, $\left(b^1, b^2\right)$ can not lie outside the outer envelope of $t^2(\cdot)$ and $t^1(\cdot)$. If $\left(b^1, b^2\right)$ lies outside the outer envelope of $t^2(\cdot)$ and $t^1(\cdot)$ and $q_2^i > 0$ for either firm, then the firm with a positive production in the second period would do better by unilaterally decreasing production in that period. If $\left(b^1, b^2\right)$ lies outside the outer envelope of $t^2(\cdot)$ and $t^1(\cdot)$ and $q_2^i = 0$, $i = 1, 2$, then either firm would do better by unilaterally decreasing production in the first period. If $\left(b^1, b^2\right)$ is such that $b^1 > N(\alpha)$ and b^2 is inside the outer envelope of $t^2(\cdot)$, then firm 2 would do better by producing $t^2(b^1) - b^2$ in the second period. Similarly, if $\left(b^1, b^2\right)$ is such that

b^1 is inside the outer envelope of $t^1(\cdot)$ and $b^2 > N(\alpha)$, then firm 1 would do better by producing $t^1(b^2) - b^1$ in the second period. If $\left(b^1, b^2\right)$ is such that $b^2 < b^1$ and $N(\beta) < b^1 < N(\alpha)$, then firm 2 would do better by producing $b^2 - b^1$ in the second period. Similarly, if $\left(b^1, b^2\right)$ is such that $b^1 < b^2$ and $N(\beta) < b^2 < N(\alpha)$, then firm 1 would do better by producing $b^1 - b^2$ in the second period. If $\left(b^1, b^2\right)$ is such that $b^1 < N(\beta)$ and b^2 is inside the outer envelope of $s^2(\cdot)$, then firm 2 would do better by producing $s^2(b^1) - b^2$ in the second period. Similarly, if $\left(b^1, b^2\right)$ is such that b^1 is inside the outer envelope of $s^1(\cdot)$ and $b^2 < N(\beta)$, then firm 1 would do better by producing $s^1(b^2) - b^1$ in the second period. That leaves points $\left(b^1, b^2\right)$ on $t^2(\cdot)$ that do not lie "between" $(N(\alpha), N(\alpha))$ and $\left((t^2)^{-1}(N(\beta)), N(\beta)\right)$ and points $\left(b^1, b^2\right)$ on $t^1(\cdot)$ that do not lie "between" $(N(\alpha), N(\alpha))$ and $\left((t^1)^{-1}(N(\beta)), N(\beta)\right)$. Suppose there is an equilibrium $\left(b^1, b^2\right)$, with $\left(b^1, b^2\right)$ in $t^2(\cdot)$ with $\left(t^2\right)^{-1}(N(\beta)) < b^1$. The fact that $\left(t^2\right)^{-1}(N(\beta)) < b^1$ implies that $b^2 < N(\beta)$. But then, the assumption that

$$U^1(S^1(\alpha,\beta), R^2(S^1(\alpha,\beta))) \leq U^1(N(\beta), N(\beta)),$$

implies

$$U^1(b^1, b^2) \leq U^1(S^1(\alpha,\beta), R^2(S^1(\alpha,\beta))) \leq U^1(N(\beta), N(\beta)),$$

that is, $\left(b^1, b^2\right)$ is not an equilibrium since firm 1 is better off by playing $q_1^1 = N(\beta)$ instead of playing any b^1 on $t^2(\cdot)$ with $\left(t^2\right)^{-1}(N(\beta)) < b^1$. Since the game is symmetric, the same arguments apply for points $\left(b^1, b^2\right)$ on $t^1(\cdot)$ with $\left(t^1\right)^{-1}(N(\beta)) < b^2$. *Q.E.D.*

Proof of Proposition 2: Let

$$E_S^{IA} = E_{S_0}^{IA} \cup E_{S_1}^{IA} \cup E_{S_2}^{IA},$$

where

$$E_{S_0}^{IA} = \left\{\left(q^1, q^2\right) : q^1 = q^2, \textit{and } N(\beta) \leq q^i \leq N(\alpha),\ i = 1, 2\right\},$$

and

$$E_{S_i}^{IA} = \left\{\left(q^1, q^2\right) : t^j(S^i(\alpha,\beta)) \leq q^j \leq \hat{q}(\alpha,\beta),\ q^i = \left(t^j\right)^{-1}(q^j)\right\},\ i = 1, 2.$$

Clearly, the method of proof used in Proposition 1 can be applied here to show that any point in $E_{S_0}^{IA}$ can be attained as a subgame perfect Nash equilibrium. So, let us show that any point in $E_{S_i}^{IA}$, $i = 1, 2$, is also a subgame perfect equilibrium. To do that we will show that the following strategies for each firm constitute a subgame perfect Nash equilibrium: $q_1^i = a^i$, where $\left(a^1, a^2\right)$ is a point in E_i^{IA} and q_2^i is given by (6). Lemma 1 shows that the strategies in the second production period are equilibrium strategies. Let us then consider the first production period. Since the game is symmetric we can focus on a single firm. So, without loss of generality, consider firm 1. If firm 1 follows the prescribed strategy it's payoff is given by

$$U^1(a^1, t^2(a^1)) = (1-\beta)\,\pi^1(a^1, t^2(a^1)) + \beta\pi^2(t^2(a^1), a^1).$$

It is obvious that firm 1 can not gain by producing more than it's prescribed first period strategy. Suppose that firm 1 deviates from it's first period strategy and produces $q_1^1 = a^1 - \varepsilon$, with $0 < \varepsilon \leq a^1 - N(\alpha)$. If that is the case, then firm 1 will not produce in the second period but firm 2 will produce $q_2^2 = t^2(q_1^1) - a^2$. So, the total output of firm 1 is $q^1 = a^1 - \varepsilon$ and the total output of firm 2 is $q^2 = t^2(a^1 - \varepsilon)$, with $0 < \varepsilon \leq a^1 - N(\alpha)$. This implies that firm 1's payoff under the deviation is given by $U^1(a^1 - \varepsilon, t^2(a^1 - \varepsilon))$. Recall that at the Stackelberg point, $\left(a^1, t^2(a^1)\right) = \left(S^1(\alpha,\beta), t^2(S^1(\alpha,\beta))\right)$, $U^1(q^1, t^2(q^1))$ attains its maximum. Since $U^1(a^1 - \varepsilon, t^2(a^1 - \varepsilon))$ is further away from the Stackelberg point than $U^1(a^1, t^2(a^1))$, it follows that

$$U^1(a^1 - \varepsilon, t^2(a^1 - \varepsilon)) < U^1(a^1, t^2(a^1)),$$

that is, firm 1 can not gain by deviating by ε, with $0 < \varepsilon \leq a^1 - N(\alpha)$, from any point $\left(a^1, t^2(a^1)\right)$ in E_1^{IA}. Now, suppose that firm 1 deviates from it's first period strategy and produces $q_1^1 = a^1 - \varepsilon$, with $a^1 - N(\alpha) < \varepsilon \leq a^1 - N(\beta)$. If that is the case, then firm 1 will not produce in the second period but firm 2 will produce $q_2^2 = q_1^1 - a^2$. So, the total output of firm 1 is $q^1 = a^1 - \varepsilon$ and the total output of firm 2 is $q^2 = a^1 - \varepsilon$, with $a^1 - N(\alpha) < \varepsilon \leq a^1 - N(\beta)$. This implies that firm 1's payoff under the deviation is given by $U^1(a^1 - \varepsilon, a^1 - \varepsilon)$. We have that

$$U^1(N(\alpha), N(\alpha)) \leq U^1(a^1 - \varepsilon, a^1 - \varepsilon) \leq U^1(N(\beta), N(\beta)).$$

The assumption

$$U^1(N(\beta), N(\beta)) = U^1\left(\hat{q}(\alpha,\beta), t^2(\hat{q}(\alpha,\beta))\right),$$

implies

$$\begin{aligned} U^1(a^1 - \varepsilon, a^1 - \varepsilon) &\leq U^1(N(\beta), N(\beta)) \\ &= U^1\left(\hat{q}(\alpha,\beta), t^2(\hat{q}(\alpha,\beta))\right) \leq U^1\left(a^1, t^2(a^1)\right), \end{aligned}$$

that is, firm 1 can not gain by deviating by ε, with $a^1 - N(\alpha) < \varepsilon \leq a^1 - N(\beta)$, from any point $\left(a^1, t^2(a^1)\right)$ in E_1^{IA}. Finally, suppose that firm 1 deviates from it's first period strategy and produces $q_1^1 = a^1 - \varepsilon$, with $a^1 - N(\beta) < \varepsilon \leq a^1$. If that is the case, then firm 1 will produce $q_2^1 = N(\beta) - q_1^1$ in period 2 and firm 2 will produce $q_2^2 = N(\beta) - q_1^2$. So, the total output of both firms will be $N(\beta)$ and this implies that firm 1's payoff under the deviation is given by $U^1(N(\beta), N(\beta))$. The assumption that

$$U^1(N(\beta), N(\beta)) = U^1\left(\hat{q}(\alpha,\beta), t^2(\hat{q}(\alpha,\beta))\right),$$

implies

$$\begin{aligned} U^1(a^1-\varepsilon, a^1-\varepsilon) &= U^1(N(\beta), N(\beta)) \\ &= U^1\left(\hat{q}(\alpha,\beta), t^2(\hat{q}(\alpha,\beta))\right) \leq U^1\left(a^1, t^2(a^1)\right), \end{aligned}$$

that is, firm 1 can not gain by deviating by ε, with $a^1 - N(\beta) < \varepsilon \leq a^1$, from any point $\left(a^1, t^2(a^1)\right)$ in E_1^{IA}.
It remains to show that there is no other subgame perfect Nash equilibrium. The method of proof used in Proposition 1 can be applied here to show that there is no other subgame perfect Nash equilibrium outside or inside the outer envelope of $t^2(\cdot)$ and $t^1(\cdot)$. That leaves points $\left(b^1, b^2\right)$ on $t^2(\cdot)$ such that $(t^2)^{-1}(N(\beta)) < b^1 < \hat{q}(\alpha,\beta)$ and points $\left(b^1, b^2\right)$ on $t^2(\cdot)$ such that $L(\alpha,\beta) < b^1$. By definition we have that $U^1\left(\hat{q}(\alpha,\beta), t^2(\hat{q}(\alpha,\beta))\right) = U^1(N(\beta), N(\beta))$. This implies that

$$U^1\left(b^1, b^2\right) < U^1(N(\beta), N(\beta)).$$

with $\left(b^1, b^2\right)$ on $t^2(\cdot)$ such that $(t^2)^{-1}(N(\beta)) < b^1 < \hat{q}(\alpha,\beta)$. However, since firm 2 is producing $q_1^2 < N(\beta)$, firm 1 can do better by producing $q_1^1 = N(\beta)$. In this case firm 1 would produce $q_2^1 = 0$ and firm 2 would produce $q_2^2 = N(\beta) - q_1^2$. This leads to a payoff of firm 1 equal to $U^1(N(\beta), N(\beta))$ which is strictly better than $U^1\left(b^1, b^2\right)$, for $\left(b^1, b^2\right)$ on $t^2(\cdot)$ such that $(t^2)^{-1}(N(\beta)) < b^1 < \hat{q}(\alpha,\beta)$. Points $\left(b^1, b^2\right)$ on $t^2(\cdot)$ such that $L(\alpha,\beta) < b^1$ are also not an equilibrium because firm 1 can do better by producing $q_1^1 = L(\alpha,\beta)$. Since the game is symmetric, the same arguments apply for points $\left(b^1, b^2\right)$ on $t^1(\cdot)$ such that $(t^1)^{-1}(N(\beta)) < b^2 < \hat{q}(\alpha,\beta)$ and points $\left(b^1, b^2\right)$ on $t^1(\cdot)$ such that $L(\alpha,\beta) < b^2$. *Q.E.D.*

Proof of Proposition 3: Let $\left(a^1, a^2\right)$ be any point in E_{HS}^{IA}. Since a^2 is a best reply to a^1, neither waiting nor any other output choice in the first production period can raise 2's payoff, and similarly for 1. Thus, any point $\left(a^1, a^2\right)$ in E_{HS}^{IA} is an equilibrium. No other outcome can be a subgame perfect equilibrium. Suppose that 1 plays $L^1(\alpha,\beta)$ in the first production period and 2 waits and then plays $t^2\left(L^1(\alpha,\beta)\right)$ in the second production period. This is not an equilibrium since the assumption that $U^1(N(\beta), N(\beta)) > U^1(L^1(\alpha,\beta), t^2(L^1(\alpha,\beta)))$ implies that 1 can do better by producing $N(\beta)$ in the first production period. Similarly, 2 playing $L^2(\alpha,\beta)$ in the first production period and 1 playing $t^1\left(L^2(\alpha,\beta)\right)$ in the second production period is not an equilibrium. A situation where 1 and 2 play $\left(b^1, b^2\right)$ in the first production period with $\left(b^1, b^2\right) \notin E_{HS}^{IA}$ is not an equilibrium since at least one of the firms is not playing her best reply to the other firm. If 1 waits, the only possible equilibrium action is 2 playing $N(\beta)$, and similarly if 2 waits. *Q.E.D.*

Proof of Proposition 4: Let

$$E_{HS}^{IA} = E_{HS_0}^{IA} \cup E_{HS_1}^{IA} \cup E_{HS_2}^{IA},$$

where

$$E_{HS_0}^{IA} = \left\{\left(q_1^1, q_1^2\right) : q_1^1 = q_1^2, \textit{and } N(\beta) \leq q_1^i \leq N(\alpha),\ i = 1, 2\right\},$$

and

$$E^{IA}_{HS_i} = \left\{ \left(q^i_1, q^j_2\right) = (L^i(\alpha,\beta), t^j(L^i(\alpha,\beta))) \right\},\ i = 1, 2.$$

Let $\left(a^1, a^2\right)$ be any point in $E^{IA}_{HS_0}$. Since a^2 is a best reply to a^1, neither waiting nor any other output choice in the first production period can raise 2's payoff, and similarly for 1. Thus, any point $\left(a^1, a^2\right)$ in $E^{IA}_{HS_0}$ is an equilibrium. Now consider the situation where firm 1 plays $L^1(\alpha,\beta)$ in the first production period and firm 2 waits and then plays $t^2(L^1(\alpha,\beta))$ in the second production period. This is equilibrium since the assumption that $U^1(N(\beta), N(\beta)) < U^1(L^1(\alpha,\beta), t^2(L^1(\alpha,\beta)))$ implies that 1 can not gain by deviating from $L^1(\alpha,\beta)$ in the first production period. Similarly, 2 playing $L^2(\alpha,\beta)$ in the first production period and 1 playing $t^1\left(L^2(\alpha,\beta)\right)$ in the second production period is an equilibrium. No other outcome can be a subgame perfect equilibrium. A situation where 1 and 2 play $\left(b^1, b^2\right)$ in the first production period with $\left(b^1, b^2\right) \notin E^{IA}_{HS_0}$ is not an equilibrium since at least one of the firms is not playing her best reply to the other firm. If 1 waits, the only possible equilibrium action is 2 playing $L^2(\alpha,\beta)$, and similarly if 2 waits. *Q.E.D.*